Physics 135-2 Placement Exam, Fall 2015
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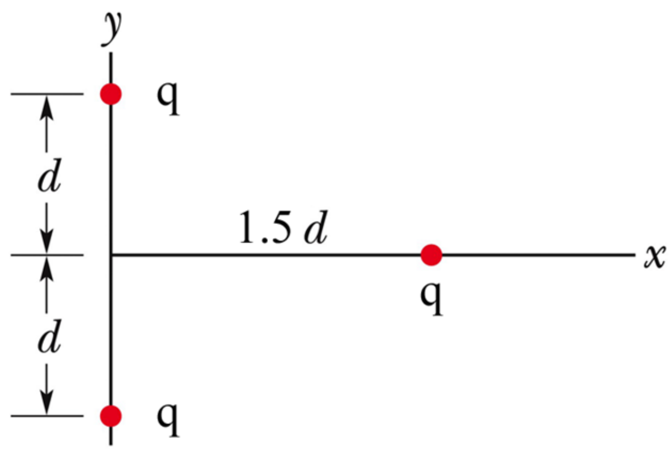
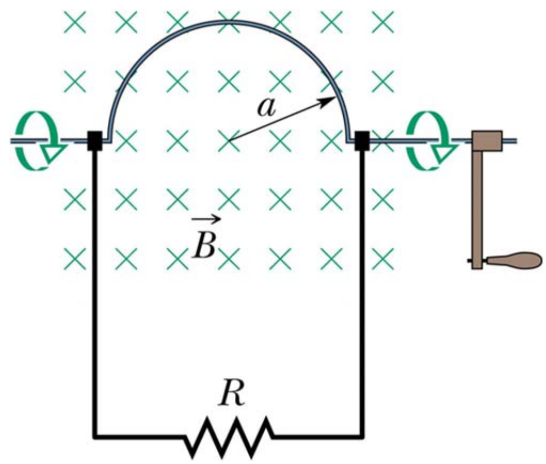


1a) (4 points) Suppose you have three charges of equal magnitude, q = +3 μC, arranged as shown at right. d = 40 nm, and the charge along the x-axis is 1.5 d= 60 nm from the y-axis.

What is the magnitude and direction of the electric force acting on the charge at the far right?

1b) (6 points) Suppose a fourth charge of q = –4 μC and mass m = 50 mg is placed at x = ∞ along the x-axis. Suppose also that this fourth charge starts out at rest, but is allowed to fall freely from x = ∞ to x = 0. At what speed will it be moving when it reaches x = 0?

2) (10 points) The arrangement at right shows a circular arc of radius a = 2 cm which is free to rotate about two pivot points when the crank is turned. (The rest of the electrical circuit does not move.) As the arc is turned, it is completely within a constant and uniform magnetic field of B = 4T which is oriented perpendicularly to the page (B goes into the page).

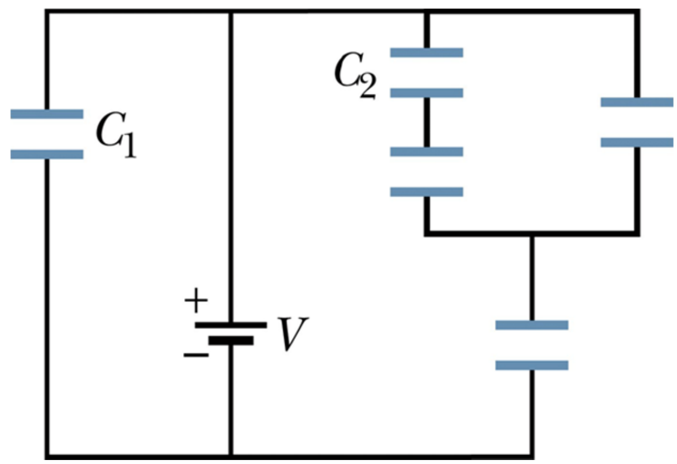


Suppose the crank is turned at a rate of 20 Hz. If we take the configuration of the circuit at t = 0 seconds to be what is shown in the diagram, then what magnitude of current will be running through the resistor R at t = 3.2 seconds? Assume R = 50 Ω.

3) Assume that you have a spherical ball of positive electrical charge which is *not* uniform. Instead, the charge has a variable radial density given by $\rho(r) = \rho_o(1 - r/a)$, where ρ_o is the density at the center of the ball, and a is the radius of the ball.

3a) (4 points) Derive the total charge of the ball in terms of ρ_o and a.
3b) (6 points) Calculate the radial electric field of the ball from $r = 0$ to $r = \infty$ in terms of ρ_o and a.

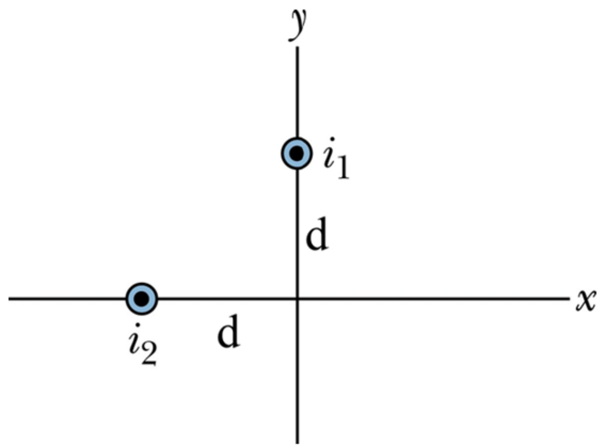
4) In the circuit at right, C_1 has a capacitance of 6 μF, C_2 has a capacitance of 4 μF, and the other three capacitors have C = 2 μF. The battery is generating V = 9 volts.



4a) (5 points) What is the total capacitance of the circuit?

4b) (5 points) What is the charge on C_2?

5) Assume that you have two electric currents i_1 and i_2 coming out of the page as shown at right. Both are at a distance d = 2 cm from the origin, and i_i = 1 amp and i_2 = 2 amps.



5a) (3 points) Write down or derive the magnitude of the magnetic field at the origin due only to i_1.

5b) (7 points) Derive the net magnetic field due to both currents at the origin. Give your answer as a magnitude plus a direction for the net magnetic field where $\theta = 0$ is the positive x-axis.

Physics 135-2 **Equation Sheet** **2018**

Kinematics: $v = v_o + at$; $\Delta x = v_o t + (½)\, a\, t^2$; $v^2 = v_o^2 + 2a(x - x_o)$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Coulomb's law: $F = k\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{|q_1||q_2|}{r^2}$

Electric Field: $\vec{E} = \frac{\vec{F}}{q_o}$ $E = \frac{1}{4\pi\varepsilon_o}\frac{|q|}{r^2}$

Gauss's Law for E: $\varepsilon_o \Phi_E = q_{enc}$ where $\Phi_E = \oint \vec{E}\cdot d\vec{A}$

Gauss's Law for B: $\Phi_B = \oint \vec{B}\cdot d\vec{A} = 0$

Potential Difference: $\Delta V = \frac{\Delta U}{q} = -\frac{W}{q} = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{l}$

Potential (point charge): $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

E from *V*: $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

Electric Potential Energy: $U = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r}$ (Pair of point charges)

Capacitors: $q = CV$ & $U_E = \frac{1}{2}CV^2 = \frac{q^2}{2C}$ (in general)

$C = \frac{\varepsilon_o A}{d}$ (parallel-plate capacitor) $E = \frac{V}{d}$

$C_{eq} = \sum_{j=1}^{n} C_j$ (parall $\frac{1}{C_{eq}} = \sum_{j=1}^{n}\frac{1}{C_j}$ (series)

Current: $i = \frac{dq}{dt} = \int \vec{j}\cdot d\vec{A}$ where $\vec{j} = (ne)\vec{v}_d$

Resistance: $R = \frac{V}{i}$ $\rho = \frac{E}{j}$ $R = \rho\frac{l}{A}$

Power: $P = iV = i^2 R = \dfrac{V^2}{R}$

Resistors: $R_{eq} = \sum_{j=1}^{n} R_j$ (series) $\dfrac{1}{R_{eq}} = \sum_{j=1}^{n} \dfrac{1}{R_j}$ (parallel)

Magnetic Force/Torque: $\vec{F}_B = q\vec{v} \times \vec{B}$ $\vec{F}_B = i\vec{l} \times \vec{B}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $|\vec{\mu}| = NiA$

Circulating Charge: $qvB = \dfrac{mv^2}{r}$ $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

Biot-Savart Law: $d\vec{B} = \dfrac{\mu_o}{4\pi} \dfrac{id\vec{l} \times \hat{r}}{r^2}$ or $dB = \dfrac{\mu_o}{4\pi} \dfrac{idl \sin\theta}{r^2}$

Ampere's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_o (i_c + i_d)$ where $i_d = \varepsilon_o \dfrac{d\Phi_E}{dt}$

Solenoid: $B = \mu_o in$ $L = \mu_o \pi n^2 R^2 l$ $n = N/l$

Faraday's Law: $\varepsilon = \oint \vec{E} \cdot d\vec{s} = -N \dfrac{d\Phi_B}{dt}$ $\Phi_B = \int \vec{B} \cdot d\vec{A}$ $\varepsilon = vBL$

Inductors: $L = \dfrac{N\Phi_B}{i}$ $\varepsilon_L = -L \dfrac{di}{dt}$ $U_B = \frac{1}{2} L i^2$ $u_B = \dfrac{B^2}{2\mu_o}$

RC Circuits:

Charging Capacitor: $q = C\varepsilon\left(1 - e^{-t/RC}\right)$ $i = \left(\dfrac{\varepsilon}{R}\right) e^{-t/RC}$

Discharging Capacitor: $q = q_o e^{-t/RC}$ $i = -\left(\dfrac{q_o}{RC}\right) e^{-t/RC}$

RL Circuits: $i = \dfrac{\varepsilon}{R}\left(1 - e^{-(R/L)t}\right)$ or $i = I_o e^{-(R/L)t}$

LC Circuits: $\omega = 2\pi f = \dfrac{2\pi}{T} = \dfrac{1}{\sqrt{LC}}$ $q = Q \cos \omega t$

Constants:
- $c = 3.00 \times 10^8$ m/s (speed of light)
- $k = 1/4\pi\varepsilon_o = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
- $\varepsilon_o = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
- $m_e = 9.11 \times 10^{-31}$ kg (electron mass)
- $m_p = 1.67 \times 10^{-27}$ kg (proton mass)
- $e = 1.6 \times 10^{-19}$ C (charge on electron)
- $\mu_o = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
- $g = 9.81 \text{ m/s}^2$

Conversions:
- $1 \text{ M}\Omega = 10^{+6}\ \Omega$
- $1 \text{ gram} = 10^{-3}$ kg
- $1 \text{ mT} = 10^{-3}$ T
- $1 \text{ kV} = 1000$ V
- $1 \text{ m} = 10^2 \text{ cm} = 10^3 \text{ mm} = 10^6\ \mu\text{m} = 10^9$ nm
- $1 \text{ eV} = 1.6 \times 10^{-19}$ J

Geometry:
- S (cylinder) = $2\pi rL + 2\pi r^2$
- S (sphere) = $4\pi r^2$
- A (circle) = πr^2
- V (cylinder) = $\pi r^2 L$
- V (sphere) = $(4/3)\,\pi r^3$
- C (circle) = $2\pi r$

Calculus: $\frac{d}{dx} \ln x = x^{-1}$ $\int \dfrac{dx}{x} = \ln x$ $\int x^n dx = x^{n+1}/_{n+1}$ $\dfrac{d}{dx} e^u = e^u \dfrac{du}{dx}$